Shewhart Control Charts
P’ Chart: Formulas

P’ Chart Formula

NHS East London NHS Foundation Trust

Data

Month	defects np)	sample size (n)
1	3852	8755
2	4100	9800
3	7083	17000
4	7339	16700
5	9406	19500
6	9310	19800
7	7250	21200
8	10400	22300
9	9250	21600
10	9950	20500
11	9846	18700
12	9854	18900
13	8034	14300
14	8162	14800
15	8122	14500
16	8200	14600
Total ∑np		130158
Total ∑n		272955

Calculation

1. First work out the pbar, using the formula below:

$$\text{pbar} = \text{CL} = \bar{p} = \frac{\sum np}{\sum n} \qquad \sum np = 130158 \quad \sum n = 272955$$

$$\bar{p} = \frac{\sum np}{\sum n} = \frac{130158}{272955} = 0.476847832 \; (0.477 \text{ to 3.d.p}) = 47.7\%$$

* Since the sample size (n) changes at each subgroup (per row), you will have to calculate the UCL and LCL for each data point. This example will just use the second row where the defect is 4100 and sample size (n) is 9800.

2. Work out the percentage (pi) of each month. For example:

$$\text{pi} = \frac{np}{n} = \frac{4100}{9800} = 0.4183673 \; (0.418 \text{ to 3.d.p}) = 41.8\%$$

3. Next, work out the standard deviation of your percentages (σPi) for each month. The formula is below:

$$\sigma Pi = \sqrt{\frac{Pbar \,*\, (1 - Pbar)}{n}} = \sqrt{\frac{0.477 \,*\, (1 - 0.477)}{9800}}$$

$$\sigma Pi = 0.0050$$

4. Next, we need to convert the percentages (pi) to Z values. This is done by using the below formula:

$$Z = \frac{Pi - Pbar}{\sigma Pi} = \quad Z = \frac{0.418 - 0.477}{0.0050} = -11.8$$

<u>It is completely fine for the Z values to be a negative number.</u>

Legend + Chart

np = number of defectives per sub group (per row)

n = sample size per sub group (per row)

$\sum np$ = sum of defects $\qquad \sum n$ = sum of sample size

pbar = CL = center line (mean)

pi = defects divided by sample size so $\text{pi} = \frac{np}{n}$

$$\sigma Pi = \sqrt{\frac{Pbar \,*\, (1 - Pbar)}{n}}$$

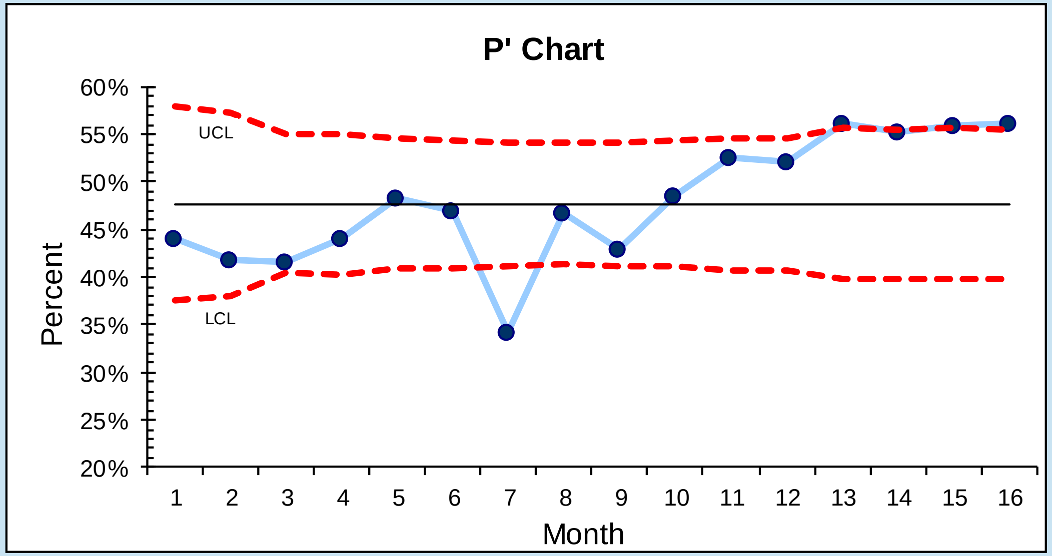
$$Z = \frac{Pi - Pbar}{\sigma Pi} \qquad \sigma Zi = \frac{MRbar}{1.128}$$

$$UCL = \text{Pbar} + 3 * (\sigma Pi) * (\sigma Zi)$$

$$LCL = \text{Pbar} - 3 * (\sigma Pi) * (\sigma Zi)$$

MRbar = Average of all MR values

MR = Difference between two consecutive data points



P’ Chart Formula

Data

Month	defects np)	sample size (n)	Z	MR	MR_ADJ
1	3852	8755	-6.9		
2	4100	9800	-11.6	4.7	4.7
3	7083	17000	-15.7	4.1	4.1
4	7339	16700	-9.7	6.0	6.0
5	9406	19500	1.5	11.2	11.2
6	9310	19800	-1.9	3.4	3.4
7	7250	21200	-39.3	37.4	
8	10400	22300	-3.1	36.2	
9	9250	21600	-14.3	11.2	11.2
10	9950	20500	2.4	16.7	16.7
11	9846	18700	13.6	11.2	11.2
12	9854	18900	12.3	1.3	1.3
13	8034	14300	20.3	8.1	8.1
14	8162	14800	18.2	2.2	2.2
15	8122	14500	20.1	1.9	1.9
16	8200	14600	20.5	0.4	0.4
Total ∑np		130158			
Total ∑n		272955			

MR_ADJ = adjusted MR after the higher values are removed and a new MR is calculated.

Average of MR = 10.4

MRbar = 6.3

Calculation

5. Next, you need to calculate the Moving Ranges (MR) of the Z values. This is done by taking the difference between consecutive values.

For example, the Z value for the first row is −6.9 and so the difference between that and the Z value for the second row (-6.9 - -11.6) is 4.7.

This needs to be done for all Z values. If there are any negative MR, just multiply them by -1.

Note: if you are doing this on excel, you may get different numbers from the calculations. This is because while excel shows a number to a decimal place, it still uses the full number. This is why the Z value in the table for month 2 is different by a 0.2 margin than the calculation done above.

6. Some of the MR values are significantly higher than the others. This is fine and they are discussed below.

7. Take the average of all of the MR values and multiply it by 3.27 (this is a standard value used). If any of the MR values are higher than this figure, then remove them from the new MR. That would mean: 10.4*3.27 = 34.008

8. We now need to calculate the standard deviation (σZi) of the new MR values. This is done using the below formula. 1.128 is a standard value used.

$$\sigma Zi = \frac{MRbar}{1.128} \quad = \quad \sigma Zi = \frac{6.3}{1.128} \qquad \sigma Zi = 5.58510638 \ (5.585 \text{ to 3.d.p})$$

Legend + Chart

np = number of defectives per sub group (per row)

n = sample size per sub group (per row)

$\sum np$ = sum of defects $\sum n$ = sum of sample size

pbar = CL = center line (mean)

pi = defects divided by sample size so pi = $\frac{np}{n}$

$$\sigma Pi = \sqrt{\frac{Pbar * (1 - Pbar)}{n}}$$

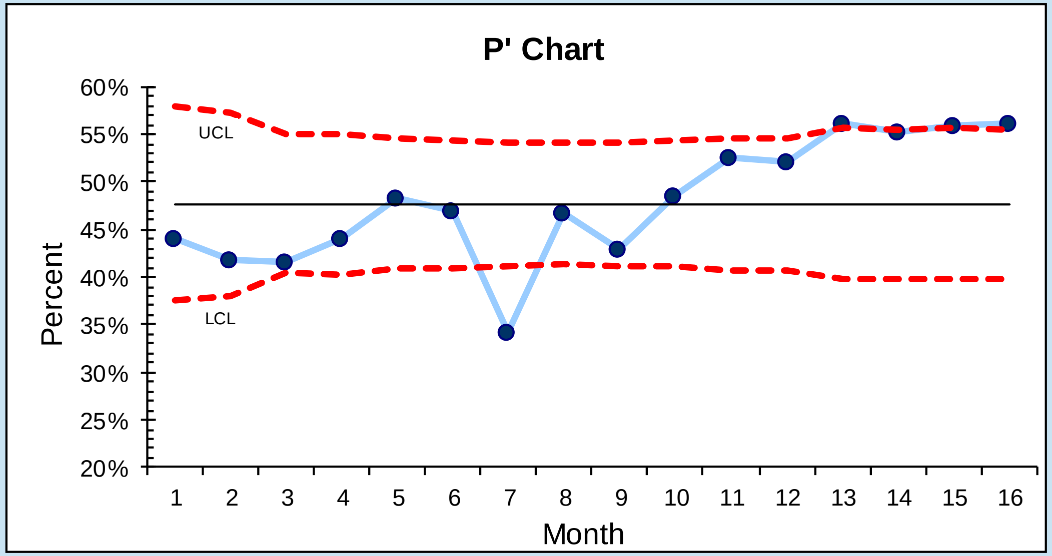
$$Z = \frac{Pi - Pbar}{\sigma Pi} \qquad \sigma Zi = \frac{MRbar}{1.128}$$

$$UCL = \text{Pbar} + 3 * (\sigma Pi) * (\sigma Zi)$$

$$LCL = \text{Pbar} - 3 * (\sigma Pi) * (\sigma Zi)$$

MRbar = Average of all MR values

MR = Difference between two consecutive data points



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Total ∑np	130158				
Total ∑n	272955				

MR_ADJ = adjusted MR after the higher values are removed and a new MR is calculated.

Average of MR = 10.4

MRbar = 6.3

Calculation

9. Finally we need to calculate the UCL and LCL using the below formulas:

Upper Control Limit

$$UCL = \text{Pbar} + 3 * (\sigma Pi) * (\sigma Zi)$$

$$UCL = 0.477 + 3 * (0.0050) * (5.585)$$

$$UCL = 0.560775\ \ (0.561 \text{ to } 3.\text{d.p}) = \mathbf{56.1\%}$$

Lower Control Limit

$$LCL = \text{Pbar} - 3 * (\sigma Pi) * (\sigma Zi)$$

$$LCL = 0.477 - 3 * (0.0050) * (5.585)$$

$$LCL = 0.393225\ \ (0.393 \text{ to } 3.\text{d.p}) = \mathbf{39.3\%}$$

10. After working out the figures for each month, you can then plot the Percentage, CL, UCL and LCL as seen on the chart

Chart Comparison

On the right, you can see a P chart as well as a P’ chart using the same data, on the left. When working with very large subgroup sizes, the P chart would not be useful it would result in tight control limits and most of the points would be outside of them, regardless of whether there were any special causes. This problem is called ‘over-dispersion’.

The P’ Chart was created as a way of dealing with this situation and is useful as the control limits appear to be more reasonable and special causes are still detected on the chart.

Charts