Shewhart Control Charts
U’ Chart: Formulas

NHS
East London
NHS Foundation Trust

U' Chart Formula

East London NHS Foundation Trust

Data

Month	Errors (np)	Per 100 Dispensed (n)
Jan-15	3852	87.55
Feb-15	6125	98.00
Mar-15	7083	176.52
Apr-15	8945	164.85
May-15	9406	195.00
Jun-15	9895	187.95
Jul-15	7250	212.36
Aug-15	10219	223.00
Sep-15	9250	216.59
Oct-15	9950	205.00
Nov-15	9846	187.00
Dec-15	9854	189.00
Jan-16	8034	143.00
Feb-16	8162	148.00
Mar-16	8122	241.26
Apr-16	8265	226.62
Total (∑np)		134258
Total (∑n)		2901.7

Calculation

1. First work out the ubar, using the formula below:

$\text{ubar} = \text{CL} = \bar{u} = \frac{\sum np}{\sum n}$ $\quad \sum np = 134258$, $\sum n = 2901.7$

$\bar{u} = \frac{\sum np}{\sum n} = \frac{134258}{2901.7} = 46.268739$ (46.3 to 1.d.p) = 46.3%

* Since the sample size (n) changes at each subgroup (per row), you will have to calculate the UCL and LCL for each data point. This example will just use the second row where the number of errors is 6125 and the total medication dispensed per 100 (n) is 98.

2. Work out the percentage (pi) of each month. For example:

$ui = \frac{np}{n} = \frac{6125}{98} = 62.5 = 62.5\%$

3. Next, work out the standard deviation of your percentages (σui) for each month. The formula is below:

$\sigma ui = \sqrt{\frac{ubar}{n}} = \sqrt{\frac{46.3}{98}} = \sigma ui = 0.687349$ (0.687 to 3.d.p)

$\sigma ui = 0.687$

4. Next, we need to convert the percentages (ui) to Z values. This is done by using the below formula:

$Z = \frac{ui - ubar}{\sigma Pi} = Z = \frac{62.5 - 46.3}{0.687} = 23.580786$ (23.581 to 3.d.p)

It is completely fine for the Z values to be a negative number.

Legend + Chart

np = number of errors per sub group (per row)

n = medication dispensed (per 100) per sub group (per row)

$\sum np$ = sum of errors $\quad \sum n$ = sum of dispensed medication (per 100)

ubar = CL = center line (mean)

ui = errors divided by dispensed medication (per 100) so ui = $\frac{np}{n}$

$\sigma ui = \sqrt{\frac{ubar}{n}}$

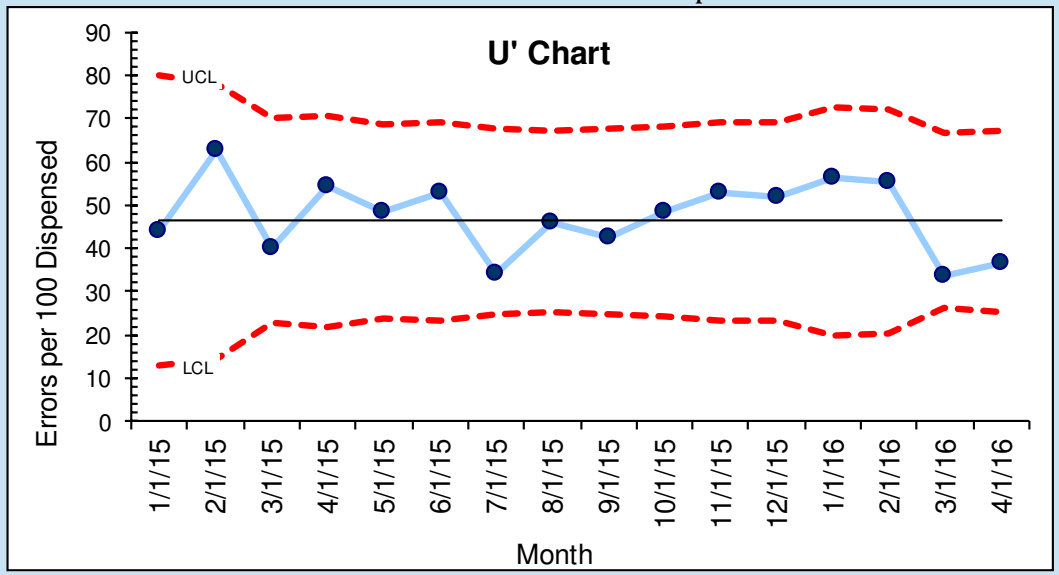
$Z = \frac{ui - ubar}{\sigma Pi}$ $\quad \sigma Zi = \frac{MRbar}{1.128}$

$UCL = \text{ubar} + 3 * (\sigma ui) * (\sigma Zi)$

$LCL = \text{ubar} - 3 * (\sigma ui) * (\sigma Zi)$

MRbar = Average of all MR values

MR = Difference between two consecutive data points



U' Chart Formula

East London NHS Foundation Trust

Data

Month	Errors (np)	Per 100 Dispensed (n)	Z	MR	MR_ADJ
Jan-15	3852	87.55	-3.12396		
Feb-15	6125	98.00	23.62226	26.74622	26.74622
Mar-15	7083	176.52	-11.9986	35.6209	35.6209
Apr-15	8945	164.85	15.08672	27.08535	27.08535
May-15	9406	195.00	4.038432	11.04828	11.04828
Jun-15	9895	187.95	12.85518	8.816752	8.816752
Jul-15	7250	212.36	-25.9838	38.83902	38.83902
Aug-15	10219	223.00	-0.97393	25.0099	25.0099
Sep-15	9250	216.59	-7.70524	6.731315	6.731315
Oct-15	9950	205.00	4.773608	12.47885	12.47885
Nov-15	9846	187.00	12.83356	8.059955	8.059955
Dec-15	9854	189.00	11.86147	0.972096	0.972096
Jan-16	8034	143.00	17.42741	5.56594	5.56594
Feb-16	8162	148.00	15.88165	1.545758	1.545758
Mar-16	8122	241.26	-28.7807	44.66231	44.66231
Apr-16	8265	226.62	-21.6842	7.096508	7.096508
Total (∑np)		134258			
Total (∑n)		2901.7			

MR_ADJ = adjusted MR after the higher values are removed and a new MR is calculated.

But there are none in this example so they can all be used.

Average of MR = 17.35

MRbar = 17.35

Calculation

Note: if you are doing this on excel, you may get different numbers from the calculations. This is because while excel shows a number to a decimal place, it still uses the full number. This is why the Z value in the table for month 2 is different by a 0.04 margin than the calculation done above.

5. Next, you need to calculate the Moving Ranges (MR) of the Z values. This is done by taking the difference between consecutive values.

For example, the Z value for the first row is –3.12 and so the difference between that and the Z value for the second row (-3.12 – 23.62) is 26.74 (this uses rounded up values).

This needs to be done for all Z values. If there are any negative MR values, just multiply them by -1.

6. Some of the MR values are significantly higher than the others. This is fine and they are discussed below.

7. Take the average of all of the MR values and multiply it by 3.27 (this is a standard value used). If any of the MR values are higher than this figure, then remove them from the new MR. That would mean: 17.35*3.27 = 56.7345

8. We now need to calculate the standard deviation (σZi) of the new MR values. This is done using the below formula. 1.128 is a standard value used.

$\sigma Zi = \frac{MRbar}{1.128}$ = $\sigma Zi = \frac{17.35}{1.128}$ $\sigma Zi = 15.381$ (15.381 to 3.d.p)

Legend + Chart

np = number of errors per sub group (per row)

n = medication dispensed (per 100) per sub group (per row)

$\sum np$ = sum of errors $\sum n$ = sum of dispensed medication (per 100)

ubar = CL = center line (mean)

ui = errors divided by dispensed medication (per 100) so ui = $\frac{np}{n}$

$\sigma ui = \sqrt{\frac{ubar}{n}}$

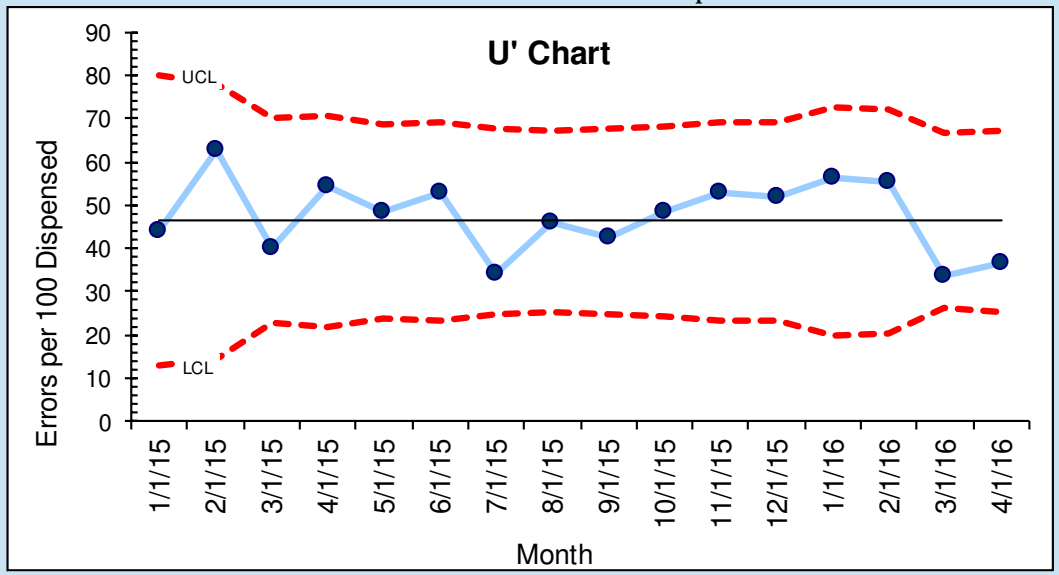
$Z = \frac{ui - ubar}{\sigma Pi}$ $\sigma Zi = \frac{MRbar}{1.128}$

$UCL = \text{ubar} + 3 * (\sigma ui) * (\sigma Zi)$

$LCL = \text{ubar} - 3 * (\sigma ui) * (\sigma Zi)$

MRbar = Average of all MR values

MR = Difference between two consecutive data points



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Average of MR = 17.35

MRbar = 17.35

Calculation

9. Finally we need to calculate the UCL and LCL using the below formulas:

Upper Control Limit

$UCL = \text{ubar} + 3 * (\sigma ui) * (\sigma Zi)$

$UCL = 46.3 + 3 * (0.687) * (15.381)$

$UCL = 78.000241 \ (78 \text{ to } 3.\text{d.p}) = 78\%$

Lower Control Limit

$LCL = \text{Pbar} - 3 * (\sigma Pi) * (\sigma Zi)$

$LCL = 46.3 - 3 * (0.687) * (15.381)$

$LCL = 14.599759 \ (14.6 \text{ to } 3.\text{d.p}) = 14.6\%$

10. After working out the figures for each month, you can then plot the Percentage, CL, UCL and LCL as seen on the chart

Chart Comparison

On the right, you can see a U chart as well as a U' chart using the same data, on the left. When working with very large subgroup sizes, the U chart would not be useful it would result in tight control limits and most of the points would be outside of them, regardless of whether there were any special causes. This problem is called '**over-dispersion**'.

The U' Chart was created as a way of dealing with this situation and is useful as the control limits appear to be more reasonable and special causes are still detected on the chart.

Charts